Evaluation of the Pion-Nucleon Sigma term from CHAOS data

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Abstract

We have reanalyzed the $\pi^{\pm}p$ scattering data at low energy in the Coulomb-nuclear interference region as measured by the CHAOS group at TRIUMF with the aim to determine the pion-nucleon σ term. The resulting value $\sigma = (43 \pm 12)$MeV is significantly lower than other recently published values.

Keywords: pion-proton scattering, derive sigma term

1. Introduction

In 2008 Ellis, Olive and Savage published a paper [1] on the elastic scattering of supersymmetric cold dark matter particles on nucleons and pointed out that the cross sections depend strongly on the value of the pion-nucleon sigma term $\sigma_{\pi N}$. This is but one example of the role of the sigma term, a concept that was introduced in chiral perturbation theory (ChPT) to measure the explicit breaking of chiral symmetry due to non-zero masses of light quarks [2]. The sigma term represents the contribution from the finite quark masses to the mass of the proton. Its value is related to the strange quark content of the nucleon. The pion-nucleon sigma term $\sigma_{\pi N}$ is also related to the value of the pion-nucleon invariant amplitude at the unphysical Cheng-Dashen point where $s - u = 0$, $t = 2m_{\pi}^2$ (Here, s, t, u are the Mandelstam variables). Consequently, its determination is mostly attempted through

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pion-nucleon scattering experiments at low energies and by spectroscopy of the hadronic shift and width in the ground state of pionic hydrogen [3].

Ellis *et al.* [1] pleaded strongly for an *experimental* campaign to better determine the pion nucleon sigma term. Indeed, due to the difficulty of experiments with low energy pions, which are notoriously plagued by pion decay and the emerging muon background, the resulting cross sections from different πN scattering experiments often do not agree with each other and the derived values of $\sigma_{\pi N}$ differ substantially, often by more than the quoted systematic errors. Ironically, however, the experimental campaign requested by the authors of [1] already existed and had been published [4] after a strong and long running effort with the CHAOS detector at TRIUMF [5], which was a dedicated detector system developed to cope with the peculiarities of low-energy pion scattering. The experiment provided differential cross sections for elastic scattering of positively and negatively charged pions off hydrogen at five energies between 19.9 and $43.3 MeV$ in fine angular steps ranging from the Coulomb-nuclear interference region to nearly 180°.

What was really missing in ref.[4] was a theoretical analysis leading to a value of $\sigma_{\pi N}$. The authors of ref.[4] merely considered the isospin-even forward scattering amplitudes $ReD^+(T_\pi)$ obtained directly from π^+p and π^-p differential cross sections and extrapolated the values to threshold ($T_\pi = 0$) using the functional forms given by the KH80 phase shifts [6] or, alternatively, from the more recent SAID FA02 phase shifts [7]. In both cases the threshold values and the related isospin-even scattering lengths turned out to be smaller than in the previous analyses. From this observation it was qualitatively concluded that the CHAOS data favour values of $\sigma_{\pi N}$ that are smaller than recently claimed [8].

It is the purpose of this paper to pursue this line of reasoning and to derive the value of the πN sigma term using the CHAOS cross sections as much as possible. This approach differs substantially from the method favored by the GWU/TRIUMF group [8] who uses the huge πN data base of SAID hoping that the errors of partially contradictory measurements are averaged out. We, instead, prefer to rely as much as possible on the results taken from the most advanced pion spectrometer to date. Furthermore, by adopting the analysis methods and the notation of the Karlsruhe group [11], we use constraints that warrant analyticity and unitarity. For a criticism of the VPI/GWU methods see [12].

2. Formalism

The πN σ term is defined as a matrix element of the quark mass term:

$$\sigma = \frac{m_u + m_d}{4m} \langle p|\bar{u}u + \bar{d}d|p\rangle,$$

where m_u and m_d are up and down quark masses, and m is the proton mass. It is an empirical measure of the chiral asymmetry generated by the up and down quark masses. The σ term may be written in the form:

$$\sigma = \frac{m_u + m_d}{4m} \frac{\langle p|\bar{u}u + \bar{d}d - 2\bar{s}s|p\rangle}{1-y},$$

where the parameter y, the strange quark content of the proton, is defined as:

$$y = \frac{\langle p|\bar{s}s|p\rangle}{\langle p|\bar{u}u + \bar{d}d|p\rangle}.$$

In a one loop calculation of chiral perturbation theory (ChPT) Gasser *et al.* [9] obtained:

$$\sigma = \frac{35.5 \pm 5}{1-y} \text{ MeV}.$$

This relation provides a simple way to calculate the strange quark content of the proton from the known value of the σ term. The low energy theorem [10], derived within perturbation chiral symmetry, relates the so called experimental πN sigma term Σ to the isospin-even scattering amplitude D^+ at the Cheng-Dashen point $\nu = 0$, $t = 2m_\pi^2$:

$$\Sigma = F_\pi^2 \bar{D}^+(\nu = 0, t = 2m_\pi^2).$$

Here $F_\pi^2 = 92.4$ MeV^2 is the pion decay constant. We use the notation from ref. [11]. The pion mass is denoted by m_π, the nucleon mass by m, and s, u, t are Mandelstam variables and $\nu = (s-u)/4m_\pi$ The amplitude $\bar{D}^+$ is defined in terms of the πN invariant amplitudes A and B: $D^+ = A^+ + \nu B^+$. $\bar{D}^+$ denotes the D^+ amplitude from which the pseudo vector Born term is subtracted: $\bar{D}^+ = D^+ - D^+_{Np\nu}$. Within the framework of ChPT Gasser *et al.* [2] obtained:

$$\sigma = \Sigma - \Delta_\sigma - \Delta_R,$$

$$\Delta_\sigma = (15.2 \pm 0.4)\text{MeV},\ \Delta_R = 0.35\text{MeV}.$$

Since the Cheng-Dashen point is outside the physical region, one has to perform an analytic continuation of the $\bar{D}^+$ amplitude to this point. Mandelstam analyticity, unitarity and crossing symmetry of the πN invariant amplitudes are very strong constraints when analyzing experimental data or when performing an analytic continuation of the invariant amplitudes outside the physical region. The most frequently used method for that purpose is the application of dispersion relations along different curves in the Mandelstam plane. It is important to stress that use of dispersion relations as a method of analytic continuation outside the physical region assumes input from the whole energy region in the physical πN channel. Using results from phase-shift analysis (PWA) in dispersion relations implies the use of a whole body of pion-nucleon data. Two different methods, both based on dispersion relations, are used for calculation of the $\bar{D}^+$ amplitude at the Cheng-Dashen point. In a first approach, the dispersion curves passes through the Cheng-Dashen point which allows calculating the amplitude D directly [14, 16, 17]. In a second approach [8, 13], one determines the coefficients in the Taylor expansion of $\bar{D}^+$ around the center of the Mandelstam triangle (also known as a sub-threshold expansion). Invariant amplitudes are real inside the Mandelstam triangle. The $\bar{D}^+$ amplitude is crossing symmetric and is a function of ν^2:

$$\bar{D}^+(\nu, t) = \sum_{m,n} \bar{d}^+_{mn} \nu^{2m} t^n.$$

At the Cheng-Dashen point one has:

$$\bar{D}^+(0, 2m_\pi^2) = \bar{d}^+_{00} + \bar{d}^+_{01} \cdot 2m_\pi^2 + \ldots$$

$$\Sigma = F_\pi^2 \cdot \bar{D}^+(0, 2m_\pi^2) = F_\pi^2 \cdot (\bar{d}^+_{00} + \bar{d}^+_{01} \cdot 2m_\pi^2 + \ldots)$$

$$\Sigma = \Sigma_d + \Delta_d.$$

Σ_d stands for a leading contribution, linear in t. The term Δ_d, a curvature term [2], describes contributions of higher order in t. Calculations show [2, 16] that the curvature term is determined mainly by contributions from the t-channel, and is considered as a known quantity. In our calculation

we use the value from [2] $\Delta_d = 11.9$ MeV. The coefficients $\bar{d}^+_{00}$ and $\bar{d}^+_{01}$ are given in terms of the $\bar{D}^+$ amplitude and its forward slope at the center of the Mandelstam triangle:

$$\bar{d}^+_{00} = \bar{D}^+(0,0),\ \bar{d}^+_{01} = \left.\frac{\partial \bar{D}^+(0,t)}{\partial t}\right|_{t=0}.$$

It is worthwhile to point out that for $t = 0$: $\bar{C}^+(\nu, t = 0) = \bar{D}^+(\nu, t = 0)$ and $\bar{d}^+_{00} = \bar{c}^+_{00}$, $\bar{d}^+_{01} = \bar{c}^+_{01}$. The amplitude C^+ is mentioned because it is used in our approach to extract information from the low energy data. Values of the σ term [8, 13, 14, 15, 16, 17] range from about 50 MeV to about 75 MeV, corresponding to large values of the strange quark content y ranging from 0.3 to 0.5. The results depend on the method and techniques used to extrapolate to the Cheng-Dashen point and on the input used.

3. Method

Whichever of the methods mentioned above is used, data from low energy $\pi^\pm p$ scattering assure more stable and more reliable extrapolations of the $\bar{C}^+$ amplitude to the Cheng-Dashen point. The goal of the CHAOS [4] experiment was to obtain high quality data needed for that purpose. In a CHAOS experiment $\pi^\pm p$ differential cross sections were measured at low energies and at small angles in the Coulomb-nuclear interference region, where the known Coulomb non-spin-flip amplitude G_c [6, 18] interferes with the nuclear amplitude G^+. The real part of forward nuclear amplitude $C^+(D^+)$ is obtained from the experimental scattering data using the formula:

$$ReC^+(q^2, t = 0) = \frac{4\pi\sqrt{s}}{m} ReG^+(q^2, t = 0) =$$
$$= \lim_{t \to 0} \frac{4\pi\sqrt{s}}{m} \left[\frac{\frac{d\sigma_{\pi^+ p}}{d\Omega} - \frac{d\sigma_{\pi^- p}}{d\Omega}}{4ReG_C(t)} \right] = \lim_{t \to 0} \Delta^+(t),$$

where G_C is the known Coulomb non-spin-flip amplitude. Measurements of differential cross sections for $\pi^\pm p$ scattering in the Coulomb-nuclear interference region allow a determination of the real part of the forward amplitude C^+ at five energies covered by the CHAOS experiment in the low energy region ranging from $T_\pi = 19.9$ MeV to 43.3 MeV. In a first step of our analysis the values Δ^+, derived from the CHAOS data at a given energy, are fitted

to a polynomial in t and extrapolated to the forward point $t = 0$. A robust convergence test function method [19] is used in which an additional term Φ is added to χ^2 in order to assure a soft cut off of higher terms in a polynomial expansion:

$$\chi^2 = \chi^2_{data} + \Phi.$$

Φ is of the form $\Phi = \lambda \sum_{n=0}^{\infty} a_n^2 (n+1)^3$, where λ is a smoothing parameter. Results for ReD^+ are shown in Fig.1.

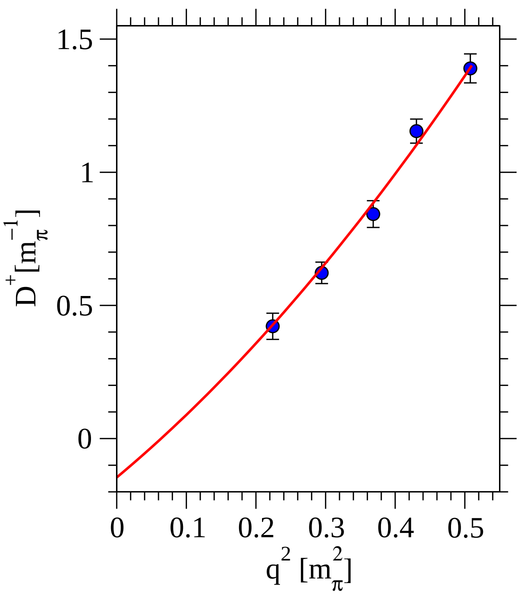

Figure 1: Data points: Real parts of the forward D^+ amplitude from the CHAOS experiment[4]. Solid curve: Fit curve as described in the text.

If the real part of the πN amplitude $C_\pm$ is consistent with the forward dispersion relations (i.e. fixed t analyticity) then, according to so called Schnitzer-Salzman plot [20, 21], the quantity:

$$Y_\pm(\omega) = ReC_\pm(\omega) - I_\pm(\omega) - C_{N\pm}(\omega)$$

is linear in ω, energy of the incoming pion in the lab. system. In the above formula $C_{N\pm}$ is a Born term and $I_\pm$ stands for an integral:

$$I_\pm(\omega) = \frac{k^2}{\pi} \int_{m_\pi}^{\infty} \left[\frac{\sigma_+(\omega')}{\omega' \mp \omega} + \frac{\sigma_-(\omega')}{\omega' \pm \omega} \right] d\omega'.$$

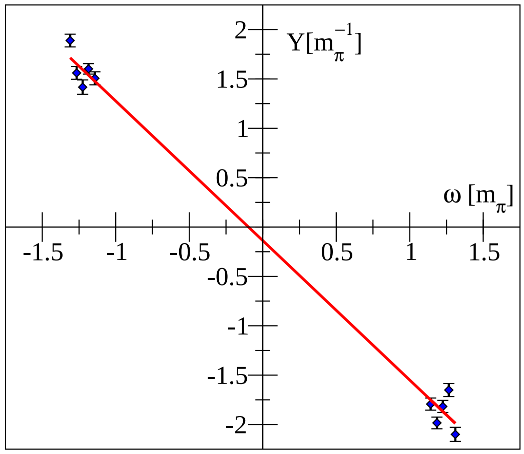

Figure 2: Schnitzer-Salzman plot for the CHAOS data (see text).

Figure 2 shows a quite satisfactory behavior of the function Y obtained from the CHAOS data. In the next step, the obtained values of $ReC^+(q^2, 0)$ are fitted to a polynomial of second order in q^2:

$$ReC^+(q^2, 0) = c_0 + c_1 q^2 + c_2 q^4, \tag{1}$$

and extrapolated to the threshold ($q^2 = 0, t = 0$). The first two coefficients c_0 and c_1 determine $ReC^+(q^2, t = 0)$ and the derivative $\frac{\partial C^+(q^2,t=0)}{\partial q^2}$ at threshold, respectively. Using the partial wave decomposition of ReC^+ and the effective range formula for partial waves $f_{l\pm} = Re\left(\frac{T_{l\pm}}{q}\right) = q^{2l}(a_{l\pm} + b_{l\pm}q^2)$ close to the threshold, one obtains the following approximation of the C^+ amplitude [11]:

$$\begin{aligned}\left(1 - \frac{t}{4m^2}\right)\frac{ReC^+(q^2, t)}{4\pi(1 + x)} \quad &= \quad a^+_{0+} + \frac{1}{2}\left(2a^+_{1+} + a^+_{1-} - \frac{a^+_{0+}}{4m^2}\right)\cdot t \\ &+ \left(b^+_{0+} + 2a^+_{1+} + a^+_{1-} + \frac{a^+_{0+}}{2m \cdot m_\pi}\right)\cdot q^2 \\ &+ \mathit{higher\ order\ terms\ in\ t\ and\ q^2}.\end{aligned}$$

Here, $a^+_{l\pm}$ are isoscalar s-and p-wave scattering lengths, b^+_{0+} is the s-wave effective range parameter [11] and $x = \frac{m_\pi}{m}$.

The second term, according to Geffens sum rule [28], is proportional to a first derivative of the C^+ amplitude with respect to q^2 at threshold:

$$\frac{1}{4\pi(1+x)}\left.\frac{\partial C^+(q^2,0)}{\partial q^2}\right|_{q^2=0} = b^+_{0+} + 2a^+_{1+} + a^+_{1-} + \frac{a^+_{0+}}{2m\cdot m_\pi}$$

$$2a^+_{1+} + a^+_{1-} = \frac{c_1}{4\pi(1+x)} - b^+_{0+} - \frac{a^+_{0+}}{2m\cdot m_\pi}. \tag{2}$$

Taking a derivative with respect to t (forward slope), one obtains:

$$\frac{1}{4\pi(1+x)}\left.\frac{\partial C^+(0,t)}{\partial t}\right|_{t=0} = \frac{1}{2}\left(2a^+_{1+} + a^+_{1-} + \frac{a^+_{0+}}{4m^2}\right) \approx \frac{1}{2}(2a^+_{1+} + a^+_{1-}). \tag{3}$$

From formulas (2) and (3) one may derive an expression relating $\left.\left(\frac{\partial C^+(q^2,t=0)}{\partial q^2}\right)\right|_{q^2=0}$ to the forward slope $\left.\left(\frac{\partial C^+(q^2=0,t)}{\partial t}\right)\right|_{t=0}$:

$$\left.\frac{\partial ReC^+(\nu=m_\pi,t)}{\partial t}\right|_{t=0} = \frac{c_1}{2} - 2\pi(1+x)\left(b^+_{0+} + \frac{a^+_{0+}}{2m\cdot m_\pi}\right).$$

The corresponding forward slope of the amplitude $\bar{C}^+$ is obtained by subtracting a pseudovector Born term:

$$\left.\frac{\partial Re\bar{C}^+(\nu=m_\pi,t)}{\partial t}\right|_{t=0} =$$
$$= \left.\frac{\partial ReC^+(\nu=m_\pi,t)}{\partial t}\right|_{t=0} - \left.\frac{\partial ReC^+_{Np\nu}(\nu=m_\pi,t)}{\partial t}\right|_{t=0}.$$

The Born term $C^+_{Np\nu}$ and its derivative are explicitly known [11]:

$$\left.\frac{\partial ReC^+_{Np\nu}(\nu=m_\pi,t)}{\partial t}\right|_{t=0} = -\frac{g^2}{4m^3}\frac{\omega}{\omega^2-\omega_B^2}\left(\omega - \frac{m_\pi^2}{\omega+\omega_B}\right).$$

In order to determine the coefficient $\bar{d}^+_{01} = \bar{c}^+_{01} = \left.\left(\frac{\partial \bar{C}^+(\nu=0,t)}{\partial t}\right)\right|_{t=0}$, one adds and subtracts the forward slope at threshold:

$$\left.\frac{\partial Re\bar{C}^+(\nu=0,t)}{\partial t}\right|_{t=0} = \left.\frac{\partial Re\bar{C}^+(\nu=0,t)}{\partial t}\right|_{t=0}$$

$$
\begin{aligned}
& - \left.\frac{\partial Re\bar{C}^{+}(\nu = m_\pi, t)}{\partial t}\right|_{t=0} + \left.\frac{\partial Re\bar{C}^{+}(\nu = m_\pi, t)}{\partial t}\right|_{t=0} \\
& = \left.\frac{\partial Re\bar{C}^{+}(\nu = m_\pi, t)}{\partial t}\right|_{t=0} + Diff
\end{aligned}
$$

$$
\begin{aligned}
\bar{c}_{01}^{+} &= \frac{c_1}{2} - 2\pi(1+x)\left(b_{0+}^{+} + \frac{a_{0+}^{+}}{4m \cdot m_\pi}\right) \\
&\quad + Diff - \left.\frac{\partial ReC_{Np\nu}^{+}(\nu = m_{\pi,t})}{\partial t}\right|_{t=0}
\end{aligned}
$$

$$
\bar{c}_{01}^{+} = \frac{c_1}{2} - 2\pi(1+x)\left(b_{0+}^{+} + \frac{a_{0+}^{+}}{4m \cdot m_\pi} + \Delta_1\right). \tag{4}
$$

Δ_1 and $Diff$ stand for a difference between the forward slope of $\bar{C}^{+}$ and C^{+} respectivelly at the center of the Mandelstam triangle ($\nu = 0, t = 0$) and at threshold ($\nu = m_\pi, t = 0$). From dispersion relations for the forward slope of C^{+} [11] one obtains the following expression:

$$
\begin{aligned}
\Delta_1 &= - \left.\frac{\partial ReC_{Np\nu}^{+}(\nu = m_{\pi,t})}{\partial t}\right|_{t=0} - \frac{2m_\pi^2}{\pi}\int_{m_\pi}^{\infty} \frac{d\omega'}{\omega'(\omega'^2 - m_\pi^2)} \\
&\quad - \frac{m_\pi}{2m}\int_{m_\pi}^{\infty} \frac{d\omega'}{\omega'^2}\frac{2\omega' + \omega}{(\omega' - m_\pi)^2} ImC^{+}(\omega', 0) \\
\Delta_1 &= - \left.\frac{\partial ReC_{Np\nu}^{+}(\nu = m_\pi, t)}{\partial t}\right|_{t=0} - I_1 - I_2.
\end{aligned} \tag{5}
$$

Both integrals are fast converging and may be accurately evaluated using results from existing partial wave analyses or, by virtue of the optical theorem, from existing data for total cross sections.

4. Results

The CHAOS data [4] consist of differential cross sections grouped in two angular regions. The first group contains data at forward lab angles, typically

below 35^0, which were covered by a range telescope for particle identification. The second group comprises all larger angles. We have performed two separate analyses of the CHAOS data. In the first analysis we have included data at forward angles only. Values of $\Delta^+(t)$ obtained from the experimental data have been fitted to a polynomial of third order in t. In the second analysis, all data were included and fitted to a polynomial of order five. In such a way two sets of values of ReC^+ were obtained from the CHAOS data and averaged. This way we deliberately increased the weight of the small angle data, keeping in mind the design features of the CHAOS detector. The resulting values of ReC^+ are nuclear. Hadronic values were calculated applying electromagnetic corrections according to the Nordita procedure from ref. [18] and used as an input for determination of the coefficients $\bar{d}^+_{00}$ and $\bar{d}^+_{01}$.

Forward dispersion relations (FDR) are an essential part of our extrapolation of the C^+ amplitude to the threshold. In the first step, values of ReC^+, obtained from the CHAOS data, combined with the once subtracted dispersion relations for the C^+ amplitude, are used to calculate its value at the threshold. The FDR for the amplitude C^+, subtracted at the threshold, read [11]:

$$
\begin{aligned}
ReC^+(\omega) &= C^+(m_\pi) + C^+_N(\omega) - C^+_N(m_\pi) \\
&\quad + \frac{2k^2}{\pi} \int_{m_\pi}^{\infty} \frac{ImC^+(\omega')}{(\omega'^2-\omega^2)(\omega'^2-m_\pi^2)} \omega' d\omega' \\
&= C^+(m_\pi) + C^+_N(\omega) - C^+_N(m_\pi) + I^+ \qquad (6) \\
C^+_N(\omega) &= -\frac{16\pi m f^2}{m_\pi^2} \frac{\omega^2}{\omega^2-\omega_B^2}, \quad \omega_B = -\frac{m_\pi^2}{2m},
\end{aligned}
$$

where k is the pion lab momentum, f^2 is the πN coupling constant and C^+_N the nucleon Born term. For each energy where ReC^+ is determined from the data, one calculates the dispersion integral and obtains the corresponding value of the subtraction constant $C^+(m_\pi)$. Partial wave analyses do not give errors of partial waves so that the error of the integral in (6) may not be given and errors quoted in our results are due only to the numerical procedure. Our calculations show that errors of the subtraction constant due to the uncertainty of dispersion integrals are negligible compared to the experimental errors.

As a result from the first step we obtain an average value of the sub-

traction constant with the corresponding error. We stress that the obtained average value is not our final result. It is used as a part of the input when calculating final values of the amplitude and its forward slope at threshold. Forward dispersion relations for the amplitude C^+ evaluated at the center of the Mandelstam triangle, give useful relations connecting values at the threshold and at the center of the Mandelstam triangle:

$$\Delta_2 = \bar{C}^+(0) - \bar{C}^+(m_\pi) = -\frac{2m_\pi^2}{\pi} \int\limits_{m_\pi}^{\infty} \frac{ImC^+(\omega')}{\omega'(\omega'^2 - m_\pi^2)} d\omega', \tag{7}$$

and

$$\bar{C}^+(0) = \bar{c}_{00}^+ = \bar{C}^+(m_\pi) + \Delta_2 = C^+(m_\pi) + 1.88f^2 + \Delta_2. \tag{8}$$

The dispersion integrals in (5) and (7) may be accurately evaluated using available scattering data. In the energy region where results from PWA exist ($k \leq k_{max}$) imaginary parts are calculated from partial waves (GWU/VPI: k_{max} = 2.6 GeV/c; Ka84: k_{max} =6.0 GeV/c). Tables of total $\pi^\pm p$ cross sections, needed to calculate ImC^+, are available up to lab momenta of k = 340 GeV/c [22]. Parametrization of the total cross sections at high energies are also available [25]. The integral in (7) is fast converging. More than 97% of its value is due to contributions below k = 2.6 GeV/c, so that uncertainties from the high energy part of the integral may be neglected. Using the GWU/VPI solution Fa08 [SAID] one obtains a value of $\Delta_2 = -1.381 m_\pi^{-1}$. Δ_1 has been calculated using the same input. Our evaluations, using several PWA solutions (Ka84, Fa02, GW06, Fa08), show that the integral I_1 is saturated already at ω_{max} which corresponds to the highest lab momentum in the GWU/VPI partial wave solution. Using the discrepancy method we found that contributions from higher energies to the integral I_1 do not exceed one percent of its value. Hence, uncertainties from high energies may be neglected. Using results from the GWU/VPI PWA solution Fa08 and tables for total cross sections at high energies, a value of Δ_1 = -133.8 GeV^{-3} = -0.364 m_π^2 was obtained. Extrapolation to threshold yields the first two coefficients in expansion (1):

$$c_0 = (-0.140 \pm 0.013) m_\pi^{-1}$$
$$c_1 = (2.146 \pm 0.187) m_\pi^{-3},$$

which are the main result of our analysis.

The only parameter left to be determined is the s- wave effective range parameter b^+_{0+}. One may follow the authors of ref. [23] and use the Karlsruhe value $b^+_{0+} = (-0.044 \pm 0.007)m_\pi^{-3}$ from ref. [11]. The very same value was obtained from partial wave relations derived from the fixed t dispersion relations [24]. Due to the fact that partial waves from partial wave relations are strictly consistent with analyticity, the method allows a reliable determination of threshold parameters. Unfortunately, like some other programs from the Karlsruhe group, Koch's program for evaluation of partial wave relations was lost. To our knowledge there are no recent determinations of the s-wave effective range parameters of such a degree of sophistication.

Until results from a new evaluation of partial wave relations become available, we rely on a simple extrapolation of the s-wave to threshold. The value we use in the present analysis is an average of the Karlsruhe value and a value obtained from the current GWU/VPI partial wave solution:

$$b^+_{0+} = (-0.050 \pm 0.004)m_\pi^{-3}.$$

Inserting values of Δ_1, Δ_2 and b^+_{0+} into formulas (2), (4), and (8) we obtain: $\bar{c}^+_{00} = (-1.3780 \pm 0.0130)m_\pi^{-1}$, $2a^+_{1+} + a^+_{1-} = (0.1994 \pm 0.0136)m_\pi^{-3}$, $\bar{c}^+_{01} = (1.0710 \pm 0.0980)m_\pi^{-3}$, and $\Sigma_d = (46.7 \pm 12.0)$ MeV. Adding the curvature term, we obtain our result for the πN σ-term as derived from the CHAOS data:

$$\Sigma = (58.6 \pm 12.0) \text{ MeV}, \quad \sigma = (43.1 \pm 12.0) \text{ MeV}.$$

As stated before, the quoted errors are mainly due to errors having their origin in the experimental data. Errors due to uncertainties of dispersion integrals, as shown by calculations, are negligible compared to the errors having their origin in the experimental data.

5. Discussion

The result obtained for the coefficient $\bar{c}^+_{00}$ corresponds to a s-wave scattering length $a^+_{0+} = (-0.0097 \pm 0.0009)m_\pi^{-1}$ which is comparable to the old Karlsruhe result [11]. The value of combination of p-waves $2a^+_{1+} + a^+_{1-}$ has not changed significantly during the last three decades. Our value $2a^+_{1+} + a^+_{1-} = (0.1994 \pm 0.0136)m_\pi^{-3}$ is in good agreement with those obtained in partial wave analyses performed during the last several years [26, 27]. It is coupled with the s-wave effective range parameter b^+_{0+} through Geffen's sum rule [28].

To study the dependence of our results on the s-wave scattering length a^+_{0+}, we alternatively used the newest result from the pionic hydrogen [3] experiment as part of the input in our fit, taking $a^+_{0+} = (0.0044 \pm 0.0022)m_\pi^{-1}$. Although the resulting fit to the data is numerically quite satisfactory, the obtained values are in strong disagreement with other low energy parameters. Namely, the value of $c_1 = (1.31 \pm 0.18)m_\pi^{-3}$ obtained in such a way is in disagreement with recently obtained values of $2a^+_{1+} + a^+_{1-}$ unless a large negative value $b^+_{0+} \leq -0.09m_\pi^{-3}$ for the s-wave effective range is allowed.

6. Conclusions

Our value of the πN σ-term, based on an analysis of the CHAOS data, is significantly lower than most values published in the last ten years. Given the systematic uncertainties of the CHAOS experiment one could not expect a precision determination of its value. Using an analysis which respects the analytic properties of the πN amplitudes we were, however, able to confirm quantitatively the conjecture of Denz *et al.* [4] that the sigma term is small, comparable to the canonical value $\sigma = 49$ MeV of Koch *et al.* [6, 14]. This is definitely much smaller than the result of the phenomenological extraction by the SAID group [8]. With no experimental facility available to improve the πN data base in the forseeable future it was important to question that result. Among other consequences we point out that, using our result, the difficulty with the unreasonably large strange quark content of proton no longer exists.

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